ITI9200 — Category Theory 2023

January 30, 2023

Why are we here?

apart because you chose to

What is category theory

You're here to learn a bit of category theory.

Category theory is a branch of Mathematics; in simple terms, a category is a structure abstracting three working assumptions of everyday mathematics:

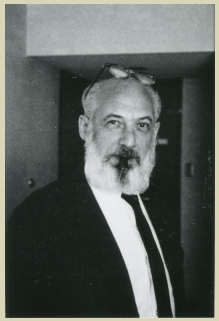
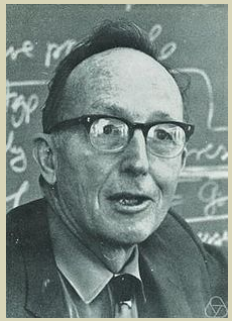
- all objects of a given type can be collected in a class; 
- such objects form coherent conglomerates, allowing for relations between structures to form; 
- far from being rare, these relational conglomerates are pretty common and arise at every corner.

What is category theory

Category theory is a young branch of Mathematics; it was first outlined in 1945, although its main ideas (structuralist philosophy) date back way more (down in the rabbithole of history of philosophy to Aristotle, for some; to I. Kant, for others; certainly F. Klein, R. Carnap, F. de Saussure,...)





What is category theory

Category theory shaped XXth century Mathematics:

- provided a foundation for algebraic geometry and topology, where it was invented, connecting previously separate areas of maths;
- provided tools to build different universes in which to interpret set- and type-theoretic foundations of mathematics;
- suggested that mathematical structures form dynamical, dialectic, interconnected organism;
- *since its very beginning* it has been used in probability theory, biology, physics, classical and quantum mechanics, computer science,. . .

Q&A

Q: What are we going to do?

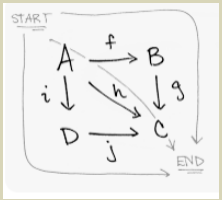
A: Category theory will follow after we have reviewed

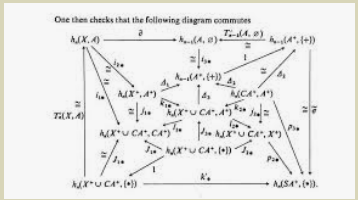
- a few facts about **ordered sets**;
- a few facts about **monoids**.

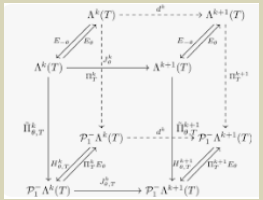
Ordered sets and monoids organize into categories, but they're categories themselves: the lesson to learn is that categories are *both*

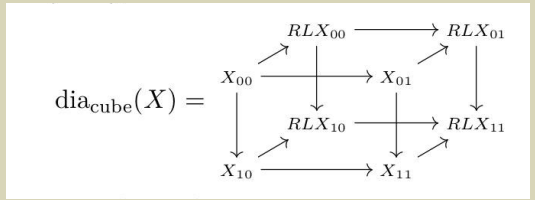
- mathematical structures, and
- universes to study the totality of structures of a given kind.

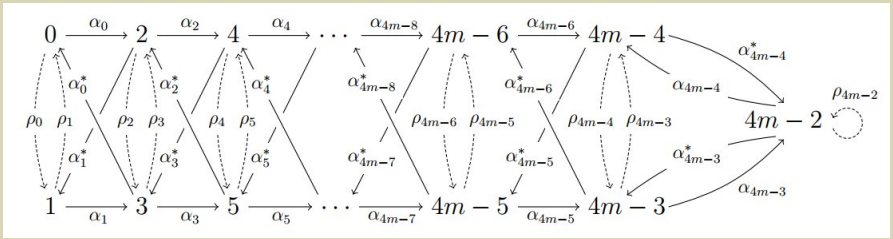
Q&A











Q: Is this course hard?

A: No. We're here to have fun, draw a lot of pictures, look for patterns; you should take this seriously but with the spirit of a stretching session rather than the Navy SEALs Hell week.

Plus, I'm here to make things easier.

Q&A

I am, by the way, the teacher of this course.

- fosco.loregian@gmail.com
- third floor of kybi

Other people that you might meet every now and then (TAing or other teachers)

- P. Sobociński https://www.ioc.ee/~pawel/
- M. Earnshaw https://www.ioc.ee/~matt/
- A. Hadzihasanovic https://www.ioc.ee/~amar/

Q&A

Q: Is category theory easy?

A: No, it's simple; it allows you to recognize common patterns between seemingly different objects.

While other people are busy doing the same thing many times, you will do it just once ('simple' comes from Latin *sēmel*: 'just once').

Life is too short to prove the same theorem twice.

Q&A

Q: Yeah but what about the exam?

A: We are in the process of discussing it. My idea: online there is a (rapidly growing) exercise set

`https://compose.ioc.ee/courses/2023/ct/exercises.pdf`

They start easy ~~to give you a false sense of secu~~because the course starts with quite simple maths. Then we take off, they take off accordingly, and reach a considerable height.

Q&A

You can

- solve as many exercises as you want (let's say: between five and ten),
- prepare a lecture on your own favourite topic (let's say: half an hour and a couple of questions).
- Using machines to solve problems is not forbidden, instead encouraged. But good luck with that.

Q&A

If you need suggestions about what you can give your presentation on, here's a few links:

- monoidal categories [,Ch 6]
- structure and semantics
- concreteness and faithful functors
- categorical logic
- 2-categories
- categories of relations / profunctors [, Ch 4]
- ends and coends
- algebraic theories
- topology and λ-calculus models
- ...

Let’s start!